Dissipative Stern-Gerlach recombination experiment

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The possibility of obtaining the initial pure state in a usual Stern-Gerlach experiment through the recombination of the two emerging beams is investigated. We have extended the previous work of Englert, Schwinger and Scully [2] including the fluctuations of the magnetic field generated by a properly chosen magnet. As a result we obtained an attenuation factor to the possible revival of coherence when the beams are perfectly recombined . When the source of the magnetic field is a SQUID(superconducting quantum interference device) the attenuation factor can be controlled by external circuits and the spin decoherence directly measured. For the proposed SQUID with dimensions in the scale of microns the attenuation factor has been shown unimportant when compared with the interaction time of the spin with the magnet.

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I. INTRODUCTION

The Stern-Gerlach experiment gives an experimental evidence of the quantum nature of the spin of a particle. When a beam of spin 1/2 particles, in the eigenstate $|+\rangle$ of S_x, goes through a variable magnetic field in the $\hat{z}$ direction and the outcoming particles are detected on a screen, we observe the presence of two distinct peaks corresponding to the spins in the positive and negative $\hat{z}$ direction.

Moreover, besides giving an evidence of the spin quantization, this example is considered the paradigm of the measurement process. The beam goes into the magnet in a pure state, for example, the eigenstate $|+\rangle_x$ of S_x, and as far as the measurement of S_z is concerned it is described as a superposition of the eigenstates of S_z. Later, as a result of the measurement process, the quantum state collapses into one of the eigenstates of the latter.

A question that arises is: could we get the initial pure state of the system if we recombine the two beams from the Stern-Gerlach experiment?

This idea of the recombination of the beams, through the so-called Stern-Gerlach Interferometer(SGI), is an old one, but has always been treated in a qualitative way [1]. One of the first quantitative studies was presented in a series of three articles [2, 3, 4], in which the authors examined the possibility of recombination and concluded that the precision of control of the magnetic field is of fundamental importance to the reconstruction of the initial pure state. However, in those articles the authors did not take into account the origin of the magnetic field and its possible sources of fluctuations.

In a more recent study [5] this problem has been revisited and the quantum nature of the magnetic field taken into account. However, only the coupling of the uniform part of field to the spin of the particle has been considered and the effects of its fluctuations on the spatial part of the wave function have been disregarded.

In another manuscript [6] a model for a dissipative Stern-Gerlach experiment was proposed without explicit reference to the origin of the fluctuations. It basically dealt with a Stern-Gerlach apparatus inside a dissipative medium and, besides, did not consider the recombination process.

In this manuscript we intend to make a fully quantum description of the magnetic field and study the effects of its unavoidable fluctuations in the spatial part of the wave function. We also pay special attention to the origin of these fluctuations by presenting a model where the magnetic field is generated by a pair of SQUIDs where the magnetic flux is known to obey a Langevin equation. We will be only interested in the spin coherence that can be measured through the mean value of S_x and arises from the off-diagonal terms of the reduced density operator of the system in the spin space.

Another point that is worth mentioning here is the fact that we will be aiming at a problem quite different from the standard one in the area of dissipative systems. Usually people want to get rid of decoherence for macroscopic quantum mechanical variables whereas we will try to increase the effect of decoherence on a microscopic variable. The reason for this is to fully test the existing theory of decoherence as unequivocally due to the sources of noise and/or dissipation known to be coupled to the system.

The paper is organized as follows: in Sec. II we present a model for the SGI where the magnetic field is generated by a pair of SQUIDs. In Sec. III we evaluate the dynamics of the reduced density operator in the spin space using the Feynman-Vernon path integral approach. In Sec. IV we analyze the coherence revival in the SGI. Finally, we summarize our results in Sec V.

II. THE STERN-GERLACH INTERFEROMETER

The SGI can be divided in two parts: the first one is responsible for separating the beams and the second one for recombining them. We will follow the basic model and

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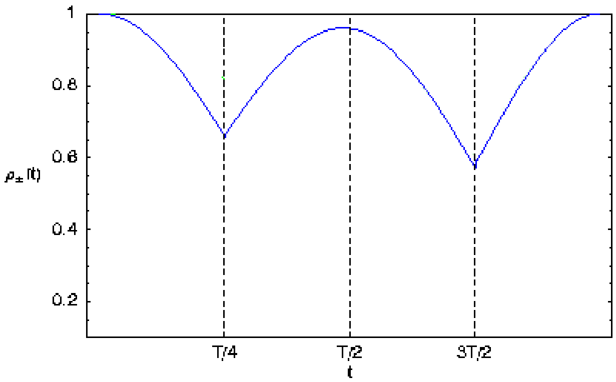

Figure 1: Loss and revival of coherence in a SGI without fluctuations.

approximations of [2], considering that the beams split in the $\hat{z}$ direction due to the magnetic field gradient which also points along this direction. As referred to a given origin, half way from two specific magnets (see arrangement below), the magnetic field should depend on y in an anti-symmetric way to enable a perfect recombination of the two beams. In order to solve the problem we will consider that the velocity in the $\hat{y}$ direction is constant which allows us to replace its position dependence by a time dependence.

Our model Hamiltonian is

$$H = \frac{p^2}{2m} - \sigma_z f\left(t\right) z \,, \tag{1}$$

where $f\left(t\right) = \mu \, \partial B/\partial z$ and μ is the magnetic moment of the particle. Here we are not considering the uniform part of the magnetic field that is only responsible for the precession the spin about the $\hat{z}$ direction and has already been treated in [5].

For a perfectly noiseless anti-symmetric field with a piecewise constant gradient we observe loss and revival of the spin coherence through the behavior of the off-diagonal elements of the reduced density operator in the spin space, as shown in Figure 1. There, T is the experiment time and the revival in the middle of the experiment is due to the recombination of the wave packets in the momentum space. The vertical lines show four different regions of a SGI. In the first part the beams are accelerated in opposite directions along $\hat{z}$. The second and third parts are due to the empty interval between the magnets where the direction of the acceleration for each beam suddenly changes. In the last part the acceleration of each beam returns to its original value and they are finally recombined as they emerge from the set up.

We now introduce the fluctuations of the magnetic field to the problem. In order to do this we will consider a SGI where each of the two magnets is formed by a pair SQUIDs arranged as shown in figure 2 . With this model we obtain a field configuration very similar to that proposed in [2] and described above.

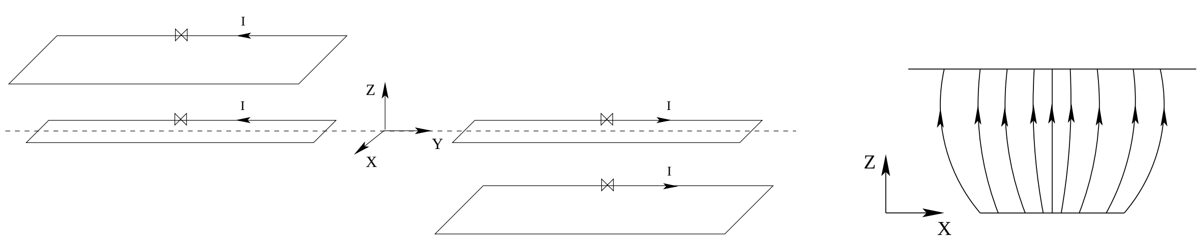

Figure 2: Lateral view of the proposed apparatus for the ISG and the magnetic field lines in a frontal view.

The total flux inside a SQUID has its dynamics described by the model Hamiltonian given by [7]

$$\begin{aligned} H_{sq+osc} &= \frac{P_\phi^2}{2C} + \frac{\Phi'^2}{2L_0} + \sum_k \left[\frac{{P_k}^2}{2m_k} \right. \\ &\quad \left. + \frac{m_k \omega_k^2}{2} \left(x_k + \frac{C_k}{m_k \omega_k^2} \Phi' \right)^2 \right] , \end{aligned} \tag{2}$$

with the spectral function $\mathcal{J}\left(\omega\right) = \frac{\pi}{2} \sum_k \frac{C_k^2}{m_k \omega_w} \delta\left(\omega - \omega_k\right) = \eta \omega$. Here Φ' is the fluctuation about one of the many metastable flux values at zero external field. We are considering that there is no tunnelling or thermal activation of the flux variable to its neighboring minima. Thus, the flux is oscillating around a local minimum close to $n\Phi_0$ and $L_0 = \left[1/L + 2\pi i_0 / n\Phi_0\right]^{-1}$ is an effective inductance, that arises from a second order expansion around the minimum of the electromagnetic potential energy. i_0 is the critical current for the SQUID. For this approximations to be valid we should be sure that we have many minima, in other words: $2\pi i_0 L/\Phi_0 \gg 1$. As a matter of fact we shall assume that each SQUID is carrying a persistent current corresponding to the metastable

minimum at $n=1$.

Here, it should be emphasized that we are only taking into account the parameters of one single SQUID of each pair, say, the smaller one. Operationally it means that the particle trajectories are always close to that same circuit over half of the experiment time. In the second half of the experiment the same applies to the second pair of SQUIDs. The presence of the larger SQUID in each pair is only to mimic the field lines of a usual Stern-Gerlach apparatus. We believe that this requirement can be dropped at the expense of using effective parameters for the coupled SQUIDs in a Hamiltonian of the same form as (2).

Therefore, we are dealing with a particle coupled to a magnetic field that is produced by a flux which, on its turn, is coupled to a bath of harmonic oscillators. Now if we write the magnetic field in terms of the flux we will have the particle indirectly coupled to the bath through the flux. This can be done if we write $\Phi = B(z)\,A(z)$ where $B(z)$ is the magnetic field at z as seen by the particles whereas $A(z)$ is an effective area crossed by the field lines through which the magnetic flux is exactly given by the total value of Φ inside the ring. Since the latter does not depend on z we have

$$B(z)\,\frac{\partial A}{\partial z} + A(z)\,\frac{\partial B}{\partial z} = 0\,, \tag{3}$$

that can be manipulated to give

$$\frac{\partial B}{\partial z} = a(z)\,\Phi\,, \quad \text{with} \quad a(z) = \frac{1}{A(z)^2}\frac{\partial A}{\partial z}\,. \tag{4}$$

Using of the last expression, the total Hamiltonian reads

$$\begin{aligned} H_{part+sq+osc} &= \frac{P^2}{2m} + \epsilon\sigma_z n\Phi_0 z + \epsilon\sigma_z\Phi' z + \frac{P_\phi^2}{2C} + \frac{\Phi'^2}{2L_0} \\ &+ \sum_k \left[\frac{{P_k}^2}{2m_k} + \frac{m_k\omega_k^2}{2}\left(x_k + \frac{C_k}{m_k\omega_k^2}\Phi'\right)^2\right], \end{aligned} \tag{5}$$

with $\epsilon = \mu a\,.$

Now we have the particle coupled indirectly to the bath of oscillators. Following the prescription of [8] we can eliminate the flux variable in the Hamiltonian and write a new one where the particle is coupled directly to new oscillators with an effective spectral function. The new Hamiltonian is

$$\tilde{H} = \frac{P^2}{2m} + \sigma_z f_0 z + \sum_k \left[\frac{{\tilde{P}_k}^2}{2\tilde{m}_k} + \frac{\tilde{m}_k\tilde{\omega}_k^2}{2}\left(\tilde{x}_k + \frac{\tilde{C}_k}{\tilde{m}_k\tilde{\omega}_k^2}\sigma_z z\right)^2\right] \tag{6}$$

For an original Ohmic spectral function, which is known to hold for SQUIDs, we obtain the following effective spectral function:

$$\mathcal{J}_{eff}(\omega) = \frac{\eta\omega}{1 + \left(\frac{\omega}{\Omega}\right)^2 + \left(\frac{\omega}{\Omega'}\right)^4} \tag{7}$$

with $\eta = \epsilon^2 L_0^2/R$, $\Omega = 1/\sqrt{L_0^2/R^2 - 2CL_0}$ and $\Omega' = 1/\sqrt{CL_0}$. Here R and C are, respectively, the resistance and capacitance of the junction of the SQUID. The effective spectral function is Ohmic with a new cutoff frequency given by the minimum of Ω and Ω'.

For typical SQUIDs we have the following values: $C \sim 10^{-12}F$, $L \sim 10^{-10}H$, $i_0 \sim 10^{-5}A$, and $R \sim 1\Omega$. To obtain $a(z)$ we will suppose a SQUID with dimensions of $\left(10^{-5}\times 10^{-3}\right)m^2$ and will estimate the total flux using the field in the middle of two infinite wires which is multiplied by the area of $10^{-8}\,m^2$. Comparing this with the flux at a distance z we can obtain an expression for $A(z)$ and then estimate $a(z)$. For $z = 10^{-3}m$ we have $a(z) = 10^{13}m^{-3}$. These specifications for the apparatus give $L_0 \sim 10^{-10}H$ and $\eta = 10^{-40}Kg\,s^{-1}$. For cooper atoms, $m = 1,8\times 10^{-25}kg$, the relaxation time, γ^{-1}, is of the order of 10^{15} seconds.

We end this section with this proposal for a model for the SGI that incorporates unavoidable sources of fluctuations of the magnetic field. In our model these fluctuations are modelled by a coupling to a bath of oscillators following [7]. Now we have to evaluate the reduced density operator of the system in the spin space to see the behavior of the spin coherence through its off-diagonal elements.

III. THE REDUCED DENSITY OPERATOR

Our model of a pair of SQUIDs generating the magnetic field for the SGI obeys the following Hamiltonian

$$H = H_0 + H_I + H_R, \tag{8}$$

with

$$H_0 = \frac{p^2}{2m} + \sigma_z f_0(t)\,z\,, \tag{9}$$

$$H_I = \sigma_z z\sum_k C_k x_x \quad \text{and} \tag{10}$$

$$H_R = \sum_k \left[\frac{p_k^2}{2m_k} + \frac{m_k\omega_k^2}{2}x_k^2 + \frac{C_k^2}{2m_k\omega_k^2}(\sigma_z z)^2\right]. \tag{11}$$

It describes the dynamics of a particle submitted to a linear potential and also coupled to a bath of oscillators both in a spin dependent way. The problem of a particle in a linear potential and coupled to a bath of oscillators has already been treated in the literature and the only difference here is the spin dependence that was absent in [6]. However, since we have only one spin operator in (8) it acts as a parameter in our problem and we will use a

slightly modified Feynman-Vernon formalism to solve it. We are not going to show all the details, of the calculation for which the reader should consult [7, 9, 10].

For separable initial conditions, where the interaction between the system and the bath is turned on at $t = 0$, one has $\langle x'\mathbf{R}'S|\,\rho(0)\,|y'\mathbf{Q}'S'\rangle = \langle x'S|\,\rho_1(0)\,|y'S'\rangle\,\langle\mathbf{R}'|\,\rho_2(0)\,|\mathbf{Q}'\rangle$ and the four elements of the reduced density operator of the system in the spin space are written as

$$\rho_{ss'}(x,y,t) = \int\int dx'dy'J_{ss'}(x,y,t;x',y',0)\,\langle x'S|\,\rho_1(0)\,|y'S'\rangle\,, \tag{12}$$

with

$$J_{ss'}(x,y,t;x',y',0) = \int\int\int d\mathbf{R}d\mathbf{R}'d\mathbf{Q}'K_s(x,\mathbf{R},t;x',\mathbf{R}',0)\,\langle\mathbf{R}'|\,\rho_2(0)\,|\mathbf{Q}'\rangle\,K^*_{s'}(y,\mathbf{R},S',t;y',\mathbf{Q}',0) \tag{13}$$

which propagates the reduced density operator in time. In this expressions $\mathbf{R},\mathbf{R}',\mathbf{Q}'$ are arbitrary configurations (N-dimensional vectors) of the bath of oscillators. Within the path integral formalism it is written as

$$J_{ss'}(x,y,t;x',y',0) = \int_{x'}^{x}Dx(t')\int_{y'}^{y}Dy(t')\,e^{\frac{i}{\hbar}\left(S_0^s[x(t')]-S_0^{s'}[y(t')]\right)}F_{ss'}[x(t'),y(t')] \tag{14}$$

with

$$\begin{aligned}F_{ss'}[x(t'),y(t')] &= \int\int\int d\mathbf{R}d\mathbf{R}'d\mathbf{Q}'\rho_2(\mathbf{R}',\mathbf{Q}',0)\int_{\mathbf{R}'}^{\mathbf{R}}D\mathbf{R}(t')\int_{\mathbf{Q}'}^{\mathbf{Q}}D\mathbf{Q}(t')\\ &\quad\times e^{\frac{1}{\hbar}\left(S_I^s[x(t'),\mathbf{R}(t')]-S_I^{s'}[y(t'),\mathbf{Q}(t')]\right)}e^{\frac{1}{\hbar}\left(S_R^s[\mathbf{R}(t')]-S_R^{s'}[\mathbf{Q}(t')]\right)}\end{aligned} \tag{15}$$

being the so-called influence functional, which contains all the influence of the bath on the system. It has been evaluated before [9] and the resulting expression is

$$F_{ss'}[x(t'),y(t')] = e^{-\frac{1}{\hbar}\Phi_{ss'}[x(t'),y(t')]}\,, \tag{16}$$

in which

$$\begin{aligned}\Phi_{ss'}[x(t'),y(t')] &= \frac{im}{2}[Sx(0)-S'y(0)]\int_0^t dt'\,\gamma(t')\,[Sx(t')-S'y(t')]+\\ &+\frac{im}{2}\int_0^t dt'\int_0^{t'}dt''\;[Sx(t')-S'y(t')]\,\gamma(t'-t'')\,[S\dot{x}(t'')+S'\dot{y}(t'')]+\\ &+\int_0^t dt'\int_0^{t'}dt''\;[Sx(t')-S'y(t')]\,\alpha_R(t'-t'')\,[Sx(t'')-S'y(t'')]\,,\end{aligned} \tag{17}$$

with

$$\gamma(t) = \frac{2}{m\pi}\theta(t)\int_0^\infty\frac{\mathcal{J}(\omega)}{\omega}\cos\omega t\;d\omega \tag{18}$$

and

$$\alpha_R\left(t'-t''\right)=\frac{1}{\pi}\int_0^\infty \mathcal{J}\left(\omega\right)\coth\left(\frac{\hbar\omega}{2KT}\right)\cos\omega\left(t'-t''\right)d\omega\,. \tag{19}$$

All we have to know is the spectral function of the bath,

$$\mathcal{J}\left(\omega\right)=\frac{\pi}{2}\sum_k\frac{C_k^2}{m_k\omega_k}\delta\left(\omega-\omega_k\right),$$

that completely characterizes it. In our proposed model we have an Ohmic case, eq [7], given by

$$\mathcal{J}\left(\omega\right)=\left\{\begin{array}{l}\eta\omega\;\; for\,\omega<\Omega\\ 0\;\;\; for\,\omega>\Omega\end{array}\right. \tag{20}$$

where we have introduced a cutoff frequency Ω. With this we obtain

$$J_{ss'}\left(x,y,t;x',y',0\right)=\int_{x'}^{x}Dx\left(t'\right)\int_{y'}^{y}Dy\left(t'\right)\,e^{\frac{1}{\hbar}\beta_{ss'}\left[x\left(t'\right),y\left(t'\right)\right]} \tag{21}$$

with

$$\begin{aligned}\beta_{ss'}\left[x\left(t'\right),y\left(t'\right)\right] \;&=\; i\int_0^t dt'\,\left[\frac{m}{2}\left(\dot{x}^2-\dot{y}^2\right)-f_0\left(t'\right)\left(Sx-S'y\right)-\frac{\eta}{2}\left(Sx-S'y\right)\left(S\dot{x}+S'\dot{y}\right)\right]+\\ &-\int_0^t dt'\int_0^{t'}dt''\,\left[Sx\left(t'\right)-S'y\left(t'\right)\right]\alpha_R\left(t'-t''\right)\left[Sx\left(t''\right)-S'y\left(t''\right)\right].\end{aligned} \tag{22}$$

Computing the diagonal ($S=S'=\pm1$) and the off-diagonals ($S\neq S'=\pm1$) terms we obtain

$$\begin{aligned}\beta_d\left[x\left(t'\right),y\left(t'\right)\right] \;&=\; i\int_0^t dt'\,\left[\frac{m}{2}\left(\dot{x}^2-\dot{y}^2\right)\mp f_0\left(t'\right)\left(x-y\right)-\frac{\eta}{2}\left(x-y\right)\left(\dot{x}+\dot{y}\right)\right]+\\ &-\int_0^t dt'\int_0^{t'}dt''\,\left[x\left(t'\right)-y\left(t'\right)\right]\alpha_R\left(t'-t''\right)\left[x\left(t''\right)-y\left(t''\right)\right]\end{aligned} \tag{23}$$

and

$$\begin{aligned}\beta_{od}\left[x\left(t'\right),y\left(t'\right)\right] \;&=\; i\int_0^t dt'\,\left[\frac{m}{2}\left(\dot{x}^2-\dot{y}^2\right)\mp f_0\left(t'\right)\left(x+y\right)-\frac{\eta}{2}\left(x+y\right)\left(\dot{x}-\dot{y}\right)\right]+\\ &-\int_0^t dt'\int_0^{t'}dt''\,\left[x\left(t'\right)+y\left(t'\right)\right]\alpha_R\left(t'-t''\right)\left[x\left(t''\right)+y\left(t''\right)\right].\end{aligned} \tag{24}$$

So we have a different dynamics for the diagonal and off-diagonal terms. Defining new coordinates $q=\left(x+y\right)/2$ and $\xi=x-y$ we decouple the $x\left(t\right)$ and $y\left(t\right)$ trajectories and get

$$J_{d(od)}\left(q,\xi,t;q',\xi',0\right)\;=\;\int_{q'}^{q}Dq\left(t'\right)\int_{\xi'}^{\xi}D\xi\left(t'\right)\;e^{\frac{1}{\hbar}\left[i\beta'_{d(od)}-\beta''_{d(od)}\right]} \tag{25}$$

with

$$\beta'_d \;=\; \int_0^t dt'\,\left[m\dot{q}\dot{\xi}-\eta\xi\dot{q}\mp f_0\left(t'\right)\xi\right], \tag{26}$$

$$\beta''_d \;=\; \int_0^t dt'\int_0^{t'}dt''\,\xi\left(t'\right)\alpha_R\left(t'-t''\right)\xi\left(t''\right), \tag{27}$$

$$\beta'_{od} \;=\; \int_0^t dt'\,\left[m\dot{q}\dot{\xi}-\eta\xi q\mp 2f_0\left(t'\right)q\right] \tag{28}$$

and

$$\beta''_{od}=4\int_0^t dt'\int_0^{t'}dt''\,q\left(t'\right)\alpha_R\left(t'-t''\right)q\left(t''\right). \tag{29}$$

Evaluating the integral in the usual way we expand it about the classical trajectory and solve the remaining functional integral to obtain

$$J_{d(od)}\left(q,\xi,t;q',\xi',0\right)=e^{\frac{1}{\hbar}\left[i\beta'^{\,class}_{d(od)}-\beta''^{\,class}_{d(od)}\right]}G\left(q,\xi,t;q',\xi',0\right) \tag{30}$$

with

$$G\left(t\right)=\frac{m\gamma e^{\gamma t}}{2\pi\hbar\sinh\gamma t}, \tag{31}$$

$$\beta_d'^{\,class} = \xi q L_-(t) + \xi' q' L_+(t) - \xi q' N(t) - \xi' q M(t) \mp X(t)\,\xi \mp Z(t)\,\xi', \quad (32)$$

$$\beta_{od}'^{\,class} = \xi q L_-(t) + \xi' q' L_+(t) - \xi q' M(t) - \xi' q N(t) \mp 2X(t)\,q \mp 2Z(t)\,q', \quad (33)$$

$$\beta_d''^{\,class} = \frac{1}{2}\left[\xi^2 A(t) + 2\xi\xi' B(t) + \xi'^2 C(t)\right], \quad (34)$$

and

$$\beta_{od}''^{\,class} = 2\left[q^2 A(t) + 2qq' B(t) + q'^2 C(t)\right]. \quad (35)$$

In the above expressions we have used $\xi(0) = \xi'$, $\xi(t) = \xi$ and the same notation holds for q. We have also defined the following functions

$$A(t) = \frac{e^{-2\gamma t}}{\sinh^2\gamma t}\int_0^t dt' \int_0^t dt''\, e^{\gamma(t'+t'')}\sinh\gamma t' \times \alpha_R(t'-t'')\sinh\gamma t'', \quad (36)$$

$$C(t) = \frac{1}{\sinh^2\gamma t}\int_0^t dt' \int_0^t dt''\, e^{\gamma(t'+t'')}\sinh\gamma(t-t') \times \alpha_R(t'-t'')\sinh\gamma(t-t''), \quad (37)$$

$$B(t) = \frac{e^{-\gamma t}}{\sinh^2\gamma t}\int_0^t dt' \int_0^t dt''\, e^{\gamma(t'+t'')}\sinh\gamma t' \times \alpha_R(t'-t'')\sinh\gamma(t-t''), \quad (38)$$

$$L_\pm(t) = m\gamma\left[\coth(\gamma t) \pm 1\right], \quad (39)$$

$$N(t) = m\gamma\frac{e^{-\gamma t}}{\sinh\gamma t}, \quad (40)$$

$$M(t) = m\gamma\frac{e^{\gamma t}}{\sinh\gamma t}, \quad (41)$$

$$X(t) = \frac{e^{-\gamma t}}{\sinh(\gamma t)}\int_0^t dt'\, f_0(t')\, e^{\gamma t'}\sinh\gamma t', \quad (42)$$

and

$$Z(t) = \frac{1}{\sinh(\gamma t)}\int_0^t dt'\, f_0(t')\, e^{\gamma t'}\sinh\gamma(t-t'). \quad (43)$$

For an initial Gaussian packet centered at the origin,

$$\psi(x) = \frac{1}{\sqrt{\sqrt{2\pi}\sigma}}\; e^{-\frac{x^2}{4\sigma^2}}, \quad (44)$$

we have

$$\rho(q', \xi', 0) = \frac{1}{\sqrt{2\pi\sigma^2}}\; e^{-\frac{q'^2}{2\sigma^2}}\; e^{-\frac{\xi'^2}{8\sigma^2}}. \quad (45)$$

The diagonal element of the reduced density operator for the system becomes

$$\rho_d(q,\xi,t) = \sqrt{\frac{\pi}{a}}G(t)\,exp\left\{-\frac{M^2}{4a\hbar^2}\left(q \mp \frac{Z}{M}\right)^2 - \xi^2\left[\frac{1}{2}\frac{A}{\hbar} + \frac{N^2\sigma^2}{2\hbar^2} - \frac{1}{4a\hbar^4}\left(\sigma^2 N L_+ - \hbar B\right)^2\right] + \right.$$
$$\left. + \frac{i}{\hbar}\left[L_- - \frac{M}{2a\hbar^2}\left(\sigma^2 N L_+ - \hbar B\right)\right] q\xi \pm \frac{i}{\hbar}\left[X + \frac{Z}{2a\hbar^2}\left(\sigma^2 N L_+ - \hbar B\right)\right]\xi\right\} \quad (46)$$

with

$$a = \frac{1}{\hbar^2}\left(\frac{L_+^2\sigma^2}{2} + \frac{\hbar C}{2} + \frac{\hbar^2}{8\sigma^2}\right), \quad (47)$$

whereas the off-diagonal element is

$$\begin{aligned}\rho_{od}(q,\xi,t) &= G(t)\sqrt{\frac{\pi}{a}}exp\left\{\left[-\frac{2\sigma^2N^2}{\hbar^2}\left(1-\frac{\sigma^2L_+^2}{2a\hbar^2}\right)+\frac{2}{\hbar}\left(\frac{B^2}{2a\hbar}-A\right)\right]q^2+\right.\\ &+\frac{i}{\hbar}\left[(\xi M\mp 2Z)\left[\frac{B}{2a\hbar}-\frac{\sigma^2NL_+}{2a\hbar^2}\right]+(\xi L_-\pm 2X)\right]q+\\ &-\left.\frac{(\xi M\mp 2Z)^2}{16a\hbar^2}-\frac{2\sigma^2L_+NB}{a\hbar^3}q^2\right\}.\end{aligned} \tag{48}$$

So we have obtained an analytical expression for the reduced density operator of the system in the spin space, when the initial spatial part of the wave function is a Gaussian centered at the origin. Now we are going to analyze this result.

IV. SPIN COHERENCE IN THE SGI

The positions of the center of the packets are given by the probability density

$$\rho_d(q,0,t)=\frac{1}{\sqrt{2\pi}\tilde{\sigma}(t)}exp-\frac{1}{2\tilde{\sigma}(t)^2}\left[q\mp\frac{Z}{M}\right]^2, \tag{49}$$

with

$$\tilde{\sigma}(t)=\frac{\hbar\sqrt{2a(t)}}{M(t)}. \tag{50}$$

So, as expected, we have a Gaussian packet with width $\tilde{\sigma}(t)$ whose center is described by

$$z(t)=\frac{Z(t)}{M(t)}=\frac{1}{2m\gamma}\int_0^t dt'\,f_0(t')\left(1-e^{-2\gamma(t-t')}\right), \tag{51}$$

which is the classical trajectory of a particle subject to a force $f_0(t)$ and to the viscous force ηv.

To see how pure the state is as it evolves in the SGI we will investigate the off-diagonal elements of the density operator in the spin space. In the high temperature limit, $KT \gg \hbar\gamma$, the functions $A(t)$, $B(t)$ and $C(t)$ can be easily evaluated and yield

$$\begin{aligned}\rho_{od}(t) &= h(t)\exp\left\{-\frac{1}{a'\hbar^2}\left[Z\left(\frac{B}{2a\hbar}-\frac{\sigma^2NL_+}{2a\hbar^2}\right)-X\right]^2\right.\\ &\left.-\frac{1}{2}\left(\frac{\Delta\tilde{z}}{\tilde{\sigma}(t)}\right)^2\right\}\end{aligned} \tag{52}$$

with

$$h(t)=\frac{M(t)}{2\hbar\sqrt{a(t)\,a'(t)}} \quad \text{and} \tag{53}$$

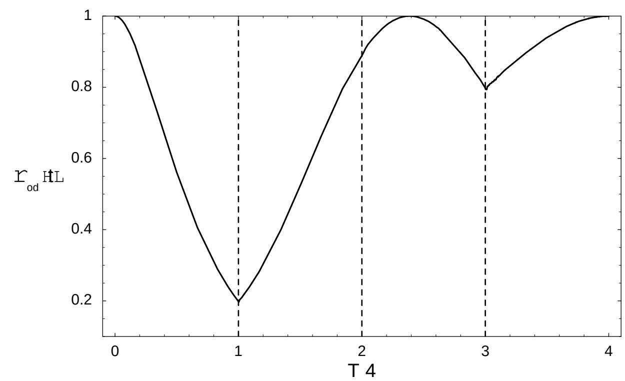

Figure 3: Behaviour of the exponent of (53), showing the possible recovery of coherence. T , the experiment time, is here of the order of $10^{-9}s$.

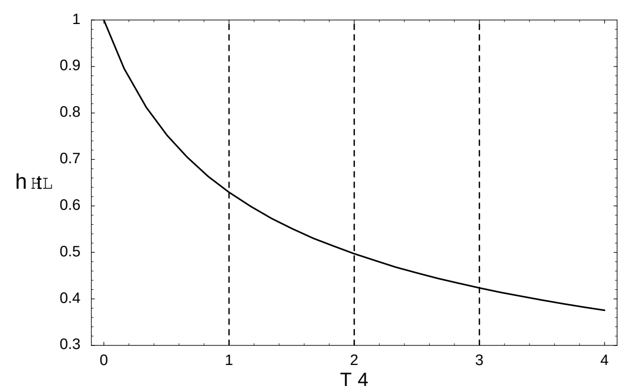

Figure 4: Factor $h(t)$ showing the irreversible loss of coherence. T is the same as above.

$$\begin{aligned}a'(t) &= \frac{2\sigma^2N^2}{\hbar^2}\left(1-\frac{\sigma^2L_+^2}{2a\hbar^2}\right)-\frac{2}{\hbar}\left(\frac{B^2}{2a\hbar}-A\right)\\ &+\frac{2\sigma^2L_+NB}{a\hbar^3}.\end{aligned} \tag{54}$$

In the non-dissipative limit, $\gamma\to 0$, $h(t)$ goes to one and the exponential recovers the non-dissipative expression for the reduced density operator. Therefore, we can say that the irreversible loss of coherence is in the factor $h(t)$. In figures 3 and 4 we show the qualitative behavior of the exponential term and the $h(t)$ factor.

In these graphs we can see that the recombination

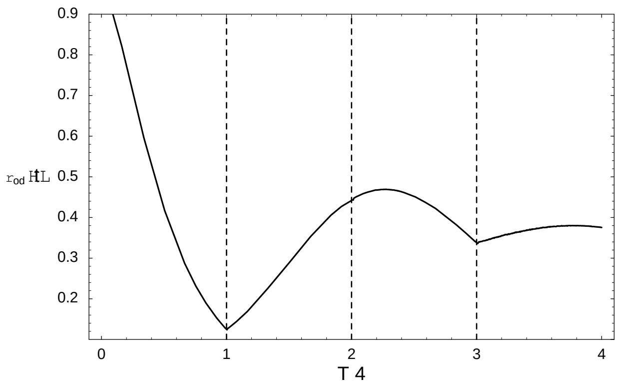

Figure 5: Behaviour of the non-diagonal element in the spin space.

could lead to a revival of coherence, but $h(t)$ continually decreases which makes this revival negligible as shown in figure 5, where, in the end, we have only 20% of coherence left.

Despite the qualitative nature of the graphs, we can say that in order to have a significant revival of coherence the time of the experiment should be much shorter than the decoherence time, which can be extracted from the expression for $h(t)$. Since this expression is very cumbersome, we have to do it grafically.

For the proposed model of a magnetic field generated by a pair of SQUIDs with dimensions of microns, we obtained $\gamma^{-1} = 10^{15}s$. These values and the other parameters already used give us the decoherence time, τ, of $10^5 s$ for temperatures close to $0.1K$.

In the usual Stern-Gerlach experiment the particles have velocities of about $1000\, m/s$ that give us an interaction time of the order of $10^{-6}s$, which is smaller than the decoherence time. So, for particles with velocities of the order of $1000\, m/s$ decoherence can not be observed and will not be important in the SGI. Therefore, in order to control the spin coherence time in an observable way we should use SQUIDs of smaller dimensions which will furnish us with shorter relaxations times. Larger SQUIDs would only make matters even worse.

Although we have only stressed the possible effect of field fluctuations on the recombination one should bear in mind that the perfect recombination is by itself a very difficult task to perform.

V. SUMMARY

In this work we have investigated the effect of spin coherence, discussing experiments with beams of particles subject to magnetic fields. We have extended the analysis of the Stern-Gerlach interferometer adding unavoidable fluctuations of the magnetic field through the coupling of the system to a bath of harmonic oscillators. The introduction of these fluctuations adds a factor to the off-diagonal elements of the reduced density operator in the spin space that decays in a time τ, the decoherence time. However we have shown that even for a very "noisy" environment provided by particularly chosen parameters for a SQUID the decoherence time is still extremely long compared to the experiment time.

When the magnetic field is generated by our proposed micrometric SQUID, we observed that dissipative effects are not important when the particles velocities are of the order of $1000\, m/s$. We believe that changing the SQUID parameters it will be possible to observe the loss of spin coherence by reducing the SQUID size. In this case we will have a model of a reservoir for a genuine microscopic quantum mechanical variable which, in our case, is the magnetic moment of an atom!

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